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LOAD DISTRIBUTION FACTOR EQUATION FOR STEEL GIRDER BRIDGES IN LRFD DESIGN

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Abstract: AASHTO LRFD 1994 Specifications introduced a new LDF equation as a result of the NCHRP12-26 project. It is considered to be a good representation of bridge behavior. However, this equation involves a longitudinal stiffness parameter, which is not initially known in design. Thus, an iterative procedure is required to correctly determine the LDF value. This need for an iterative design procedure is perceived by practicing engineers as the major impediment to widespread acceptance of the AASHTO LRFD equation. In this study, a new simplified equation based on the AASHTO LRFD formula is developed that does not require an iterative procedure. A total of 43 representative bridges are selected and analyzed using a sophisticated finite element model. The new simplified equation produces LDF values that are always conservative when compared to those obtained from the finite element analyses and are generally greater than the LDF obtained using AASHTO LRFD specification.

Key Words: Load Distribution, Steel Girder Bridge, LRFD Design, Finite Element Analysis

1. INTRODUCTION

In bridge design, the maximum moment in the girders is necessary in the determination of the bridge section. The problem is three-dimensional and involves complex behavior of load transfer from concrete slab to steel girder. The AASHTO bridge specification suggests many methods to analyze bridges, i.e., finite element analysis, grillage analysis, and a load distribution factor (LDF) equation.

The LDF equation is introduced to facilitate in determination of maximum moment in the girders. Finite element analysis (FEA) is considered to be an accurate method, but it requires much effort in data preparation, bridge modeling and analysis, and interpretation of results. With the LDF equation, the maximum moment in the girders is obtained by multiplying a moment from a one-dimensional bridge analysis by the value obtained from the LDF equation.

The wheel load distribution factor from the “S-over” equation, the AASHTO standard equation (*AASHTO 1996*), for concrete slab on steel girder bridges with two or more design lanes loaded is

$$LDF = \frac{S}{5.5} \qquad \text{(US customary unit)}$$
$$LDF = \frac{S}{1676} \qquad \text{(SI unit)} \qquad (1)$$

where S is girder spacing (ft, mm). The S-over equation, first introduced in 1930s, involves only one parameter. Although the S-over equation is simple to use, it is considered to be unsafe for some bridges and too conservative for others.

In 1994, a more accurate LDF equation was introduced in the first edition of AASHTO LRFD specification and ongoing to appear in the current AASHTO LRFD specification (*AASHTO 2004*). This equation was based on FEA and statistics. The wheel load distribution factor equation from AASHTO LRFD for concrete slab on steel girder bridges with two or more design lanes loaded is

$$LDF = 0.15 + \left(\frac{S}{3}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12Lt_s^3}\right)^{0.1} \qquad \text{(US customary unit)}$$
$$LDF = 0.15 + \left(\frac{S}{914}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{Lt_s^3}\right)^{0.1} \qquad \text{(SI unit)} \qquad (2)$$

where *S* is girder spacing (ft, mm), *L* is span length (ft, mm), $K_g = n(I+Ae^2)$ is longitudinal stiffness (in^4, mm^4), t_s is slab thickness (in, mm), *n* is modular ratio between steel and concrete, *I* is girder stiffness (in^4, mm^4), *A* is girder area (in^2, mm^2), and *e* is eccentricity between centroids of girder and slab (in, mm).

The AASHTO LRFD equation is considered to represent well the actual behavior of bridges (*Zokaie 2000, Zokaie et.al. 1991*). However, since the equation requires parameters that are not known until girder selection, an iterative design procedure is necessary. These parameters are longitudinal stiffness, K_g, and slab thickness, t_s.

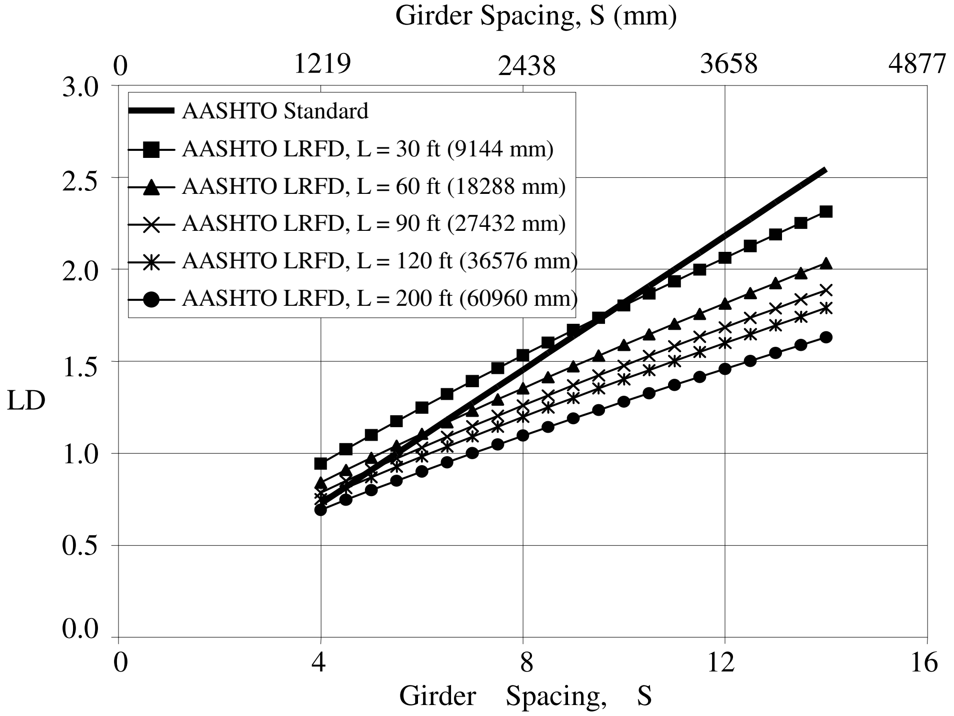

Figure 1. Comparison of LDFs from AASHTO Standard and AASHTO LRFD equations (Unitless stiffness term in AASHTO-LRFD is set to unity)

Figure 1 shows the comparison between LDFs from the AASHTO Standard and AASHTO LRFD equations. Note that the third term in the AASHTO LRFD equation, $K_g/12Lt_s^3$ for US customary units and K_g/Lt_s^3 for SI units, is assumed to be equal to unity as recommended for a first trial in design. For bridges in general, this term ranges from 0.85 to 1.10. Compared to the AASHTO LRFD, the AASHTO Standard equation tends to give unconservative LDF when bridge span length and girder spacing are relatively small, and gives overly conservative LDF when bridge span length and girder spacing are relatively large. Although the AASHTO Standard equation is simple, it yields inaccurate LDF values. Conversely, the AASHTO LRFD equation produces accurate results, but it is considered to be cumbersome in practice.

The objective of this study is to propose a new simplified LDF equation for concrete slab on steel girder bridges that captures the load distribution behavior but does not require an iterative design procedure. In the proposed specification, the parameters in the AASHTO LRFD equation that introduce the need for iteration are eliminated. The new simplified equation is intended to be at least as conservative as the LRFD equation. The scope of the research is limited to concrete slab on steel I-girder bridges.

2. METHODOLOGY

The approach adopted in this work includes the development of a reliable three-dimesional finite element model and the postulation and verification of the new simplified LDF equation. The new simplified LDF equation is formulated based on the AASHTO LRFD equation. The formulation involves the elimination of the parameters that create the need for an iterative design procedure. Various finite element models for slab-on-girder bridges are studied. An appropriate model is selected and employed to verify the new simplified LDF equation, ensuring its safety.

3. SIMPLIFIED LDF FORMULATION

The AASHTO-LRFD formula contains four parameters: girder spacing, span length, longitudinal stiffness, and slab thickness. In the formulation of the new simplified LDF equation, the sensitivity of the LDF to each parameter is studied. The goal is to eliminate parameters for which the LDF is not as sensitive as others, as well as those that require iterative design procedure.

According to the sensitivity studies performed both in the NCHRP 12-26 Project (*Zokaie et. al. 1991*) and in the present work, girder spacing (S) was the most sensitive parameter in computation of the LDF. Span length (L) is the next most sensitive parameter and longitudinal stiffness (K_g) somewhat influences the LDF. The LDF appears to be least sensitive to changes in slab thickness (t_s).

Based on the results from the sensitivity studies, some parameters were kept and others eliminated from the new simplified LDF equation. Girder spacing and span length are kept since from the sensitivity study these parameters are identified as having the most influence on the LDF value. The slab thickness of bridges is typically 8 inches (200 mm), so this parameter is eliminated and the thickness is assumed to be 8 inches (200 mm).

Meanwhile, the longitudinal stiffness parameter is also to be eliminated. The longitudinal stiffness parameter is introduced in AASHTO-LRFD to increase the accuracy of the equation (*Zokaie et. al. 1991*). Since the section properties of the girder are not known prior to determination of the LDF, the third term in the LRFD equation, which contains the longitudinal stiffness parameter, is assumed to be a unit value in the first calculation. After the girder section is determined, the LDF equation is reevaluated to check the strength criterion. This iterative procedure is cumbersome since redesign may be required. The intention is to eliminate the longitudinal stiffness parameter and the need for an iterative procedure in the LDF calculation.

The longitudinal stiffness parameter (K_g) is found to be related to the span length parameter (L). In Figure 2, the longitudinal stiffness of the bridge database from the NCHRP project 12-26 are plotted versus the span length. The general trend of the relationship is that K_g increases as L increases, but the data are scattered in relatively wide range. One of the reasons may be the bridge engineer's preference on the selection of girder dimensions.

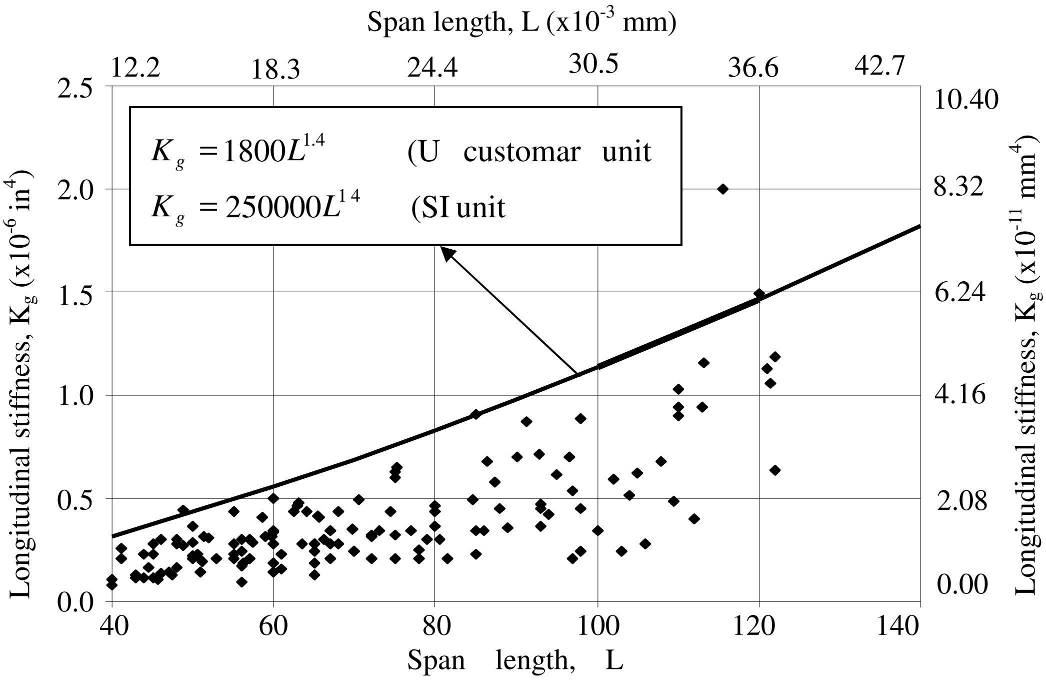

Figure 2. Trend line for relationship between longitudinal stiffness and span length

An upper bound relationship between K_g and L can be defined by the polynomial trendline regression. This trendline covers all bridges in a conservative manner, as shown in Figure 2, and it is given by

$$\begin{aligned} K_g &= 1800L^{1.4} && \text{(US customary unit)} \\ K_g &= 250000L^{1.4} && \text{(SI unit)} \end{aligned} \tag{3}$$

With this relationship almost all of the bridges are conservatively represented. The only one bridge that does not fall below the trend line will be verified later by FEA. This K_g-L relationship will be used in the construction of the new simplified equation.

Since the LRFD equation is presumed to be accurate, the postulation of the new LDF equation is constructed based on the current AASHTO-LRFD equation (Equation 1). With a slab thickness equal to 8 inches (200 mm) and the relationship between the longitudinal stiffness and the span length from Equation (3), the new simplified formula for the wheel load distribution factor of concrete slab on steel girder bridges with two or more design lanes loaded is derived as

$$LDF = 0.15 + \frac{S^{0.8}}{2.19L^{0.16}} \quad \text{(US customary unit)}$$
$$LDF = 0.15 + \frac{S^{0.8}}{85L^{0.16}} \quad \text{(SI unit)} \qquad (4)$$

where S is the girder spacing (feet, mm) and L is the span length (feet, mm). The advantage of the new simplified equation is that it includes the most influential parameters (S, L, and K_g). The girder spacing and span length parameters are incorporated explicitly, while the longitudinal stiffness is built in implicitly through the relationship to the span length. In this fashion, the iterative procedure in the LDF determination is eliminated.

Table 1. Load Distribution Formulas for Steel Girder Bridges (US customary unit*)

Specification	Basic LDF Formula	Skew correction factor
AASHTO Standard	$\frac{S}{5.5}$	N/A
AASHTO LRFD	$0.15+\left(\frac{S}{3}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12Lt_s^3}\right)^{0.1}$	for $\theta \geq 30°$ $1-0.25\left(\frac{K_g}{12Lt_s^3}\right)^{0.25}\left(\frac{S}{L}\right)^{0.5}(\tan\theta)^{1.5}$
Simplified	$0.15+\frac{S^{0.8}}{2.19L^{0.16}}$	for $\theta \geq 30°$ $1-0.184\left(\frac{S^{0.5}}{L^{0.4}}\right)(\tan\theta)^{1.5}$

* Units of S, L, K_g, and t_s are ft, ft, in^4, and in, respectively.

Table 2. Load Distribution Formulas for Steel Girder Bridges (SI unit**)

Specification	Basic LDF Formula	Skew correction factor
AASHTO Standard	$\frac{S}{1676}$	N/A
AASHTO LRFD	$0.15+\left(\frac{S}{914}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{Lt_s^3}\right)^{0.1}$	for $\theta \geq 30°$ $1-0.25\left(\frac{K_g}{Lt_s^3}\right)^{0.25}\left(\frac{S}{L}\right)^{0.5}(\tan\theta)^{1.5}$
Simplified	$LDF=0.15+\frac{S^{0.8}}{85L^{0.16}}$	for $\theta \geq 30°$ $1-0.104\left(\frac{S^{0.5}}{L^{0.4}}\right)(\tan\theta)^{1.5}$

** Units of S, L, K_g, and t_s are mm, mm, mm^4, and mm, respectively.

The base equation of Simplified LDF formula does not take into account the skew correction. The simplified LDF should be reduced by the skew correction factor as identified in Table 1 for US customary units and Table 2 for SI units, respectively. The skew correction factor for the new Simplified equation is also derived from one in the AASHTO-LRFD specification by using the Kg-L relationship (Equation 3).

4. FINITE ELEMENT ANALYSIS

4.1 Finite Element Model

Several finite element bridge models were studied using the commercial finite element analysis software, ABAQUS (*ABAQUS Inc. 2001*). It has been concluded that the eccentric beam model gives as close to real idealization as possible while retaining simplicity for practical use (*Chan and Chan 1999*). The eccentric beam model ensures full composite action between the deck slab and the girders. This model utilizes the non-composite section properties of two elements to model composite action by imposing the rigid links (ABAQUS MPC) between the centroid of the girder and the mid-surface of the slab as shown in Figure 3.

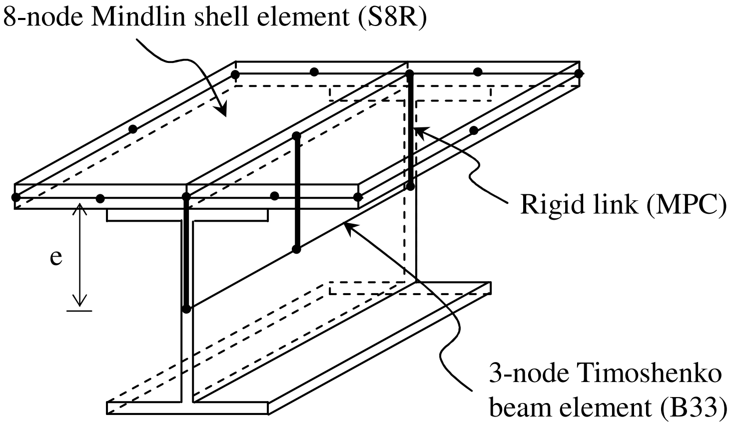

Figure 3. Finite Element Eccentric Beam Model for Slab on Girder Bridges

In the present study, the concrete slab deck is modeled by 8-node Mindlin shell elements (ABAQUS S8R), which combine plate bending elements and membrane stretch elements, while the steel girder is modeled by 3-node Timoshenko beam elements (ABAQUS B33). This element selection was made in order to ensure that the FE model retains compatibility along the element boundaries. The bearings are modeled by assigning boundary conditions to the zero-dimensional elements at their real location. For the simply supported beam, rotations along all directions are allowed in order to simulate the simply supported structure. Minimum restraints are assigned for longitudinal and transverse movement while vertical restraint is placed at the supports. Kinematic constraints are also supplied to nodes between the girders and the deck.

Consistent with the AASHTO specification, the LDF is determined by back-calculation from the maximum moment and the moment from one-dimensional beam analysis. The one-dimensional beam analysis is analogous to the bridge configuration. The length of beam is the same as the bridge span length. However, the loading on the beam analysis is one line of wheel loads placed at the position that produces the maximum moment. The LDF can be calculated using the moment in the girder section divided by the maximum moment from the one-dimensional beam analysis.

4.2 Verification of Finite Element Model

The finite element model developed for this study is verified with field-testing conducted at the University of Michigan (*Eom and Nowak 2001*). The tested bridge is a simple span located on Stanley Road over I-75 in Flint, Michigan. The span length is 126 ft (38,405 mm). There are 7 girders with girder spacing of 7.25 ft (2,210 mm) and an overhanging width of 2.45 ft (747 mm). The slab thickness is 8 in (203 mm). The cross section of the tested bridge is shown in Figure 4. Strain gauges were installed at the bottom flanges of the girders. All strains were measured along the centerline of bridge span. The load distribution factors were then calculated from the strains at the specific girders. The test load for field-testing is the Michigan three-unit, 11-axle truck. The load test was performed with the truck at crawl speed to produce the maximum static strain at the steel girders.

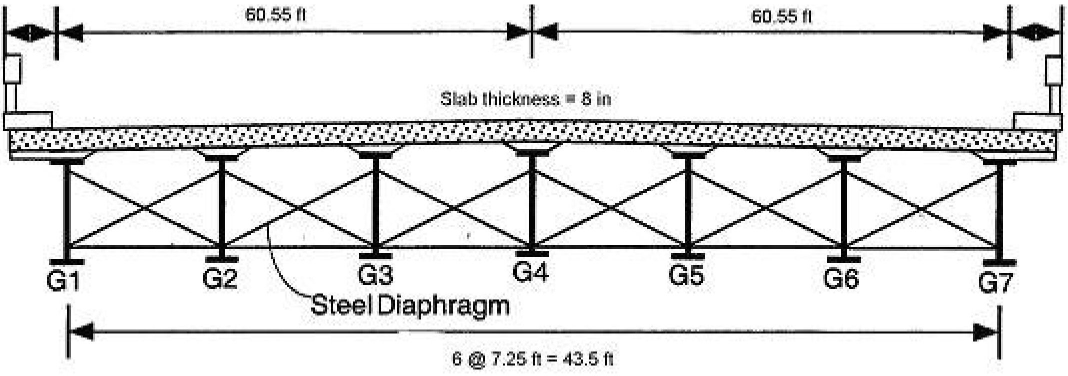

Figure 4. Cross Sectional View of Tested Michigan Bridge (*Eom and Nowak, 2001*)

The FE model used for this study is verified with the experimental results. More specifically, the calculated LDFs are compared to those obtained from the test results. These findings are shown in Figure 5. As can be seen, good agreement between measured and calculated values is observed in all girders. Therefore, the FE results may be regarded as the "exact" results.

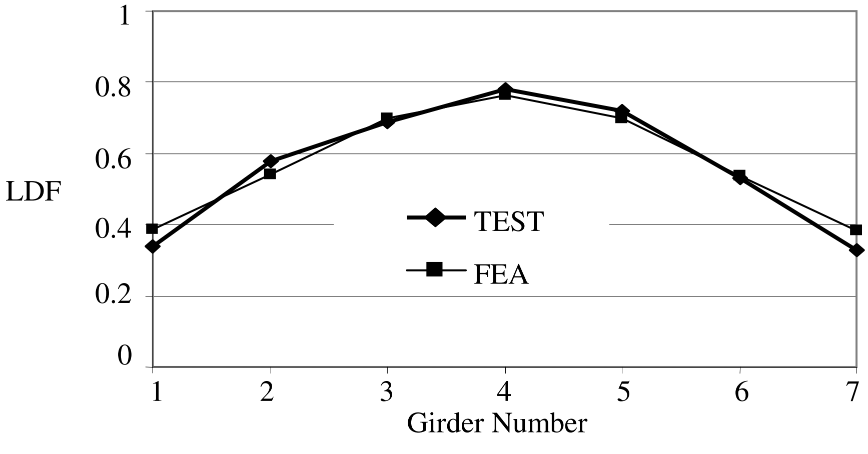

Figure 4. LDF Comparison between Finite Element Analysis and Test Result

5. LOAD DISTRIBUTION FACTOR COMPARISONS

The load distribution factors obtained using the proposed simplified equations are compared with those obtained using AASHTO LRFD. They are also compared with those resulting from the FEA. The definitions of the terms used in the LDF comparison are given in Tables 1 and 2.

The ratio of Simplified LDF to LRFD LDF for different span lengths is shown in Figure 6. Each data point represents a real bridge from the NCHRP 12-26 databases. Simplified LDFs are greater than LRFD LDFs except for one data point which is located under the dotted line. It is, as predicted, a data point for the bridge beyond the upper limit defined in equation (3). Although at first glance it seem to be unconservative LDF value, further studies using the developed FE model reveal that the simplified LDFs are greater than those obtained using FEA results. Thus, it can be ascertained that the new equation always produces conservative LDFs.

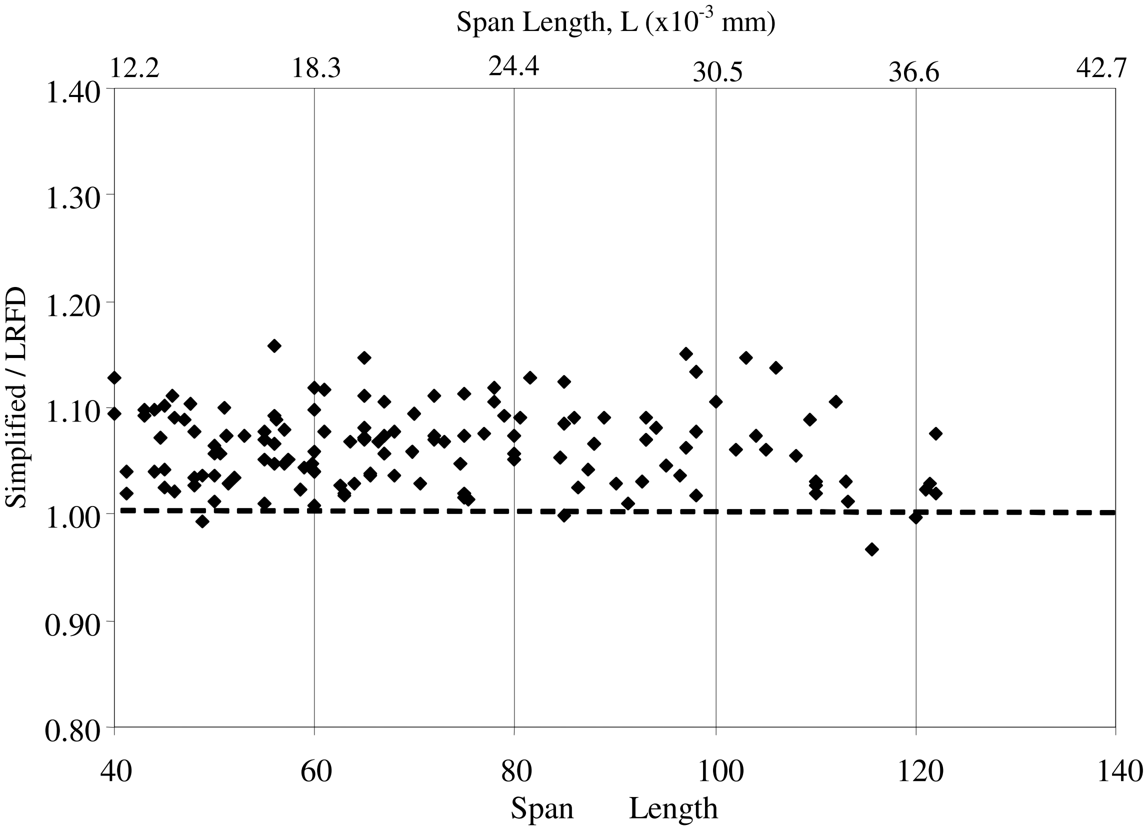

Figure 5. LDF Comparisons between Simplified Equation and LRFD Equation

In Figure 7 and Figure 8 the Simplified LDF and the LRFD LDF are compared to the FEM LDF for positive moment and negative moment, respectively. Each of the data points is an LDF ratio for one of the representative bridges. As mentioned earlier, the FEM LDF is considered to be the “exact” LDF. It is observed that LRFD LDFs are generally conservative, but for some bridges unconservative results are obtained. Simplified LDFs, on the other hand, are always conservative.

The Simplified LDF may be greater than the LRFD LDF by up to 16 percent depending on the bridge geometry. However, the Simplified LDF is always conservative, unlike the

LRFD LDF. The Simplified and LRFD LDF are greater than the Standard LDF when span length and girder spacing are relatively small. In other words, the Standard LDF is not conservative in those cases. Therefore, the simplified formula is similar to the LRFD formula in its ability to represent the load distribution. Finally, the new simplified equation is simple, requires no iteration, and produces LDF values that are at least as conservative as the ones obtained by the LRFD equation.

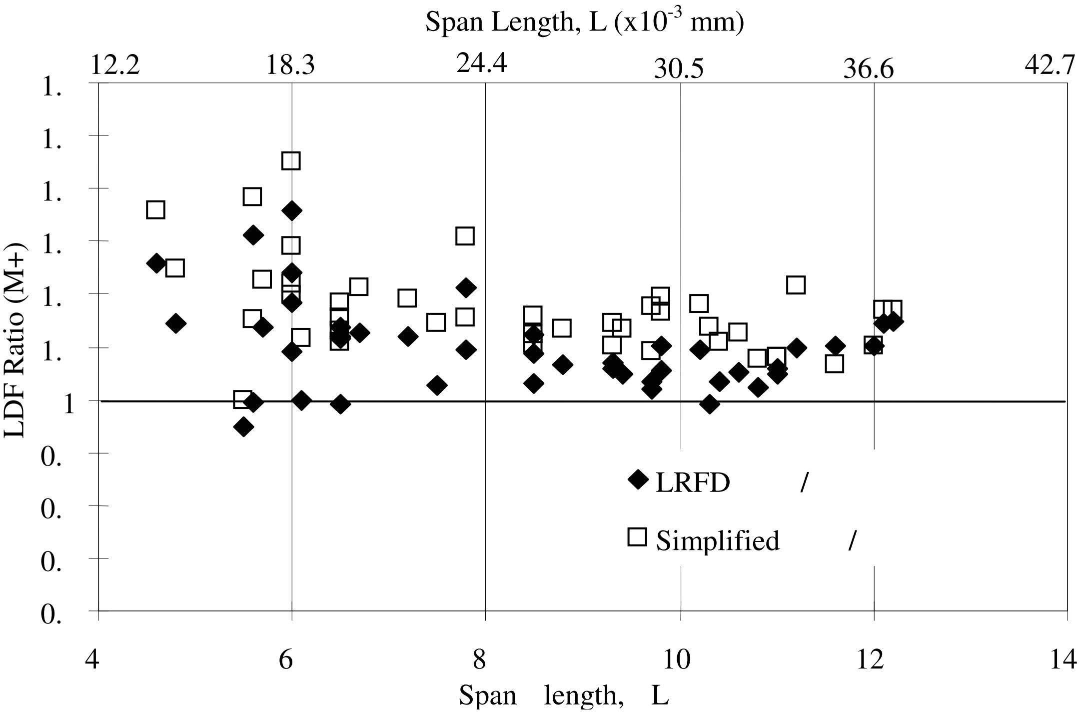

Figure 6. Positive Moment LDF Comparisons

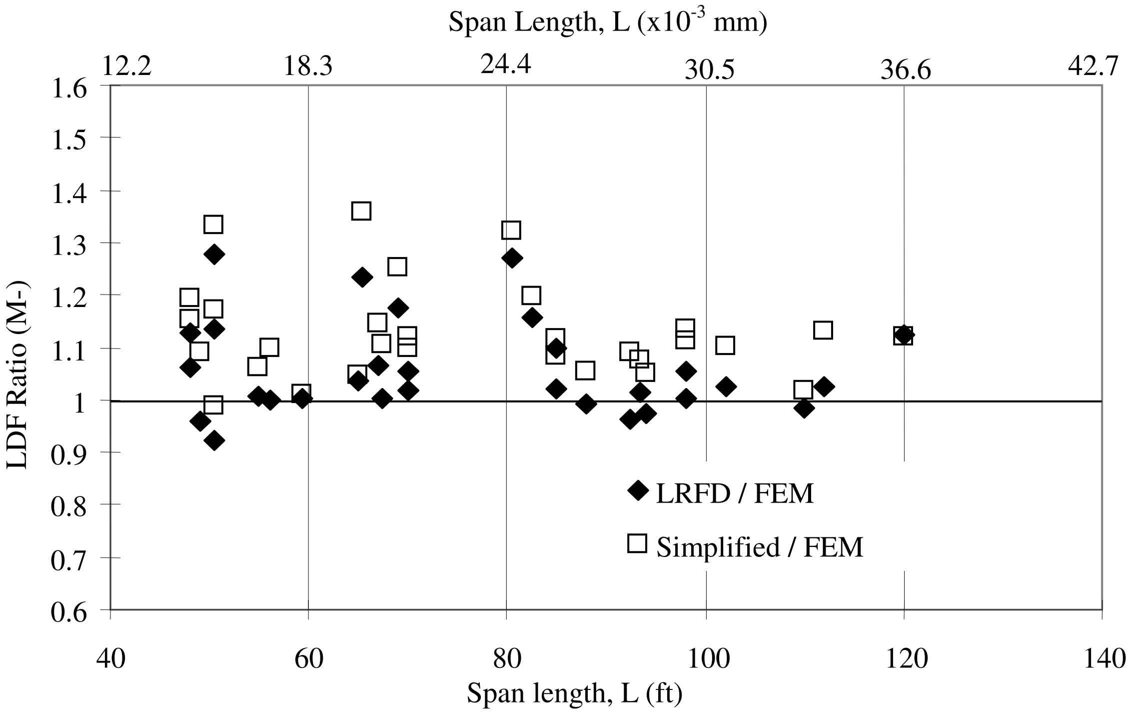

Figure 7. Negative Moment LDF Comparisons

6. CONCLUSIONS

The main objective of this research is to provide the simplest, yet most sufficiently accurate equation for calculation of load distribution. A new simplified wheel load distribution factor (LDF) equation is postulated. This equation is based on the current AASHTO LRFD. However, the longitudinal stiffness parameter (K_g) and the slab thickness parameter (t_s) that appear in the LRFD equation are implicitly embedded in the simplified expression. This eliminates the iterative procedure introduced by the LRFD formula. The accuracy and applicability of the simplified equation are demonstrated by comparisons to the AAHSTO-Standard, AASHTO-LRFD, and AASHTO level-three analysis, namely finite element (FE) analysis. It is shown that the simplified equation provides a simple yet safe specification for LDF calculation.

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